SRI VENKATESWARA UNIVERSITY
B.A. / B.Sc. DEGREE COURSE IN MATHEMATICS
III SEMESTER

(Under CBCS W.E.F. 2021-22)

Course Outcomes:

After successful completion of this course, the student will be able to;

1. Acquire the basic knowledge and structure of groups, subgroups and cyclic groups.
2. Get the significance of the notation of a normal subgroups.
3. Get the behavior of permutations and operations on them.
4. Study the homeomorphisms and isomorphism's with applications.
5. Understand the ring theory concepts with the help of knowledge in group theory and to prove the theorems.
6. Understand the applications of ring theory in various fields.

Course Syllabus:

UNIT – I (12 Hours)

GROUPS :

Binary Operation – Algebraic structure – semi group-mooned – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT – II (12 Hours)

SUBGROUPS :

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition- examples-criterion for a complex to be a subgroup. Criterion for the product of two subgroups to be a subgroup-Union and Intersection of subgroups.

Co-sets and Lagrange's Theorem :

Cossets Definition – properties of Cossets–Index of a subgroups of a finite groups–Lagrange's Theorem.

UNIT –III (12 Hours)

NORMAL SUBGROUPS :

Definition of normal subgroup –Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group –quotient group – criteria for the existence of a quotient group.

HOMOMORPHISM :

Definition of homomorphism – Image of homomorphism - elementary properties of homomorphism – Isomorphism – auto orphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem of Homomorphism.

UNIT – IV (12 Hours)

PERMUTATIONS AND CYCLIC GROUPS :

Definition of permutation – permutations multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Cyclic Groups :- Definition of cyclic group – elementary properties – classification of cyclic groups.

UNIT – V (12 Hours)

RINGS: Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws Rings, Integral Domains, Division Ring and Fields, The characteristic of a ring - The characteristic of an Integral Domain, The characteristic of a Field. Sub Rings, Ideals.

Co-Curricular Activities (15 Hours)

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

Text Book :

A text book of Mathematics for B.A. / B.Sc. by B.V.S.S. SARMA and others, published by Scand & Company, New Delhi.

Reference Books:

Abstract Algebra by J.B. Farleigh, Published by Nervosa publishing house.

1. Modern Algebra by M.L. Hanna.
2. Rings and Linear Algebra by Pundit & Pundit, published by Pragmatic Prakasham.

CBCS/ SEMESTER SYSTEM

(W.E.F. 2020-21 Admitted Batch)

B.A./B.Sc. MATHEMATICS

III SEMESTER

COURSE-III, ABSTRACT ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks

5 X 5 M=25 M

1. Define group. Give an example of a non-abelian group.
2. Prove that cancellation laws holds in a group.
3. If H and K are two subgroups of a group G, then prove that HK is a subgroup if and only if HK=KH
4. If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup.
5. Examine whether the following permutations are even or odd
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 8 & 9 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$$
6. Prove that a group of prime order is cyclic.
7. Prove that the characteristic of an integral domain is either prime or zero.
8. Prove that a field has no proper ideals.

SECTION - B

Answer ALL the questions. Each question carries TEN marks.

5 X 10 M = 50 M

9. a) Show that the set of n^{th} roots of unity forms an abelian group under multiplication.

(Or)

b) Prove that a finite semi-group with cancellation laws is a group.

10 a) Prove that union of two subgroups is also a subgroup if and only if one is contained in the other.

(Or)

b) State and prove Langrage's theorem for finite groups.

11. a) Prove that a subgroup H of a group G is a normal subgroup of G if the product of two right cosets of H in G is again a right coset of H in G.

(Or)

b) State and prove fundamental theorem of homomorphism of groups.

12. a) Let S_n be the symmetric group on n symbols and let A_n be the group of even permutations. Then show that A_n is normal of S_n and $o(A_n)=\frac{n!}{2}$.

(Or)

b) Prove that every subgroup of cyclic group is cyclic.

13. a) Prove that every finite integral domain is a field.

(Or)

b) Define an ideal of a ring. Prove that intersection of two ideals of a ring is also an ideal.